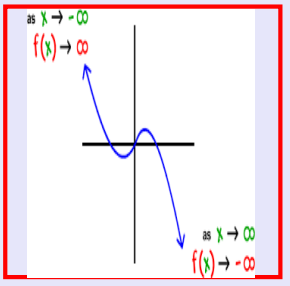
Math 245

Spring 2022

Lecture 26



Quadratic Equation: $ax^2+bx+c=0$, $a\neq 0$

Quadratic Formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Discriminant: b^2-4ac

Proof of Quadratic Formula:

$ax^2+bx+c=0$; $a\neq 0$

Divide by a to make the Lead. Coef. 1

$\dfrac{a}{a}x^2+\dfrac{b}{a}x+\dfrac{c}{a}=0 \Rightarrow x^2+\dfrac{b}{a}x+\dfrac{c}{a}=0$

Move the constant to the right hand side

$x^2+\dfrac{b}{a}x = -\dfrac{c}{a}$

Now make a perfect-square for the Left Side

$\dfrac{1}{2}\cdot\dfrac{b}{a}=\dfrac{b}{2a}$, $\left(\dfrac{b}{2a}\right)^2=\dfrac{b^2}{4a^2}$

$x^2+\dfrac{b}{a}x+\dfrac{b^2}{4a^2} = \dfrac{b^2}{4a^2}-\dfrac{c}{a}$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c \cdot 4a}{a \cdot 4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Now use the Square-Root Method

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}} \qquad x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

The Quadratic formula $\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

Solve $(x+1)(5x-2) = 6$ by the quadratic formula. Write in $ax^2 + bx + c = 0$ form

$$5x^2 - 2x + 5x - 2 - 6 = 0$$

$$5x^2 + 3x - 8 = 0$$

$a = 5$, $b = 3$, $c = -8$

$$b^2 - 4ac = 3^2 - 4(5)(-8) = 9 + 160 = 169$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-3 \pm \sqrt{169}}{2(5)} = \frac{-3 \pm 13}{10}$$

$$x = \frac{-3+13}{10} = \frac{10}{10} = 1$$

$$x = \frac{-3-13}{10} = \frac{-16}{10} = \frac{-8}{5}$$

$$\left\{\frac{-8}{5}, 1\right\}$$

The sum of squares of two cons. odd integers is 34.

Find all such odd integers.

$x, x+2$ has to be odd.

$$x^2 + (x+2)^2 = 34$$
$$x^2 + (x^2 + 4x + 4) = 34$$
$$2x^2 + 4x + 4 - 34 = 0$$
$$2x^2 + 4x - 30 = 0$$
Divide by 2 $\Rightarrow$ $x^2 + 2x - 15 = 0$

Let's solve this by Completing the Square method.

$$x^2 + 2x + 1 = 15 + 1$$
$$(x+1)^2 = 16$$
by S.R.M.
$$x+1 = \pm\sqrt{16}$$
$$x = -1 \pm 4$$
$x = -1+4 = 3$
$x = -1-4 = -5$

x	$x+2$
3	5
-5	-3

Final Ans: 3 & 5 or -5 & -3

The area of rectangular room is $36\ m^2$. The length is 1 m longer than twice its width.

Find its dimensions.

$A = 36$

4m by 9m

$x > 0$

4m

$A = 36\ m^2$

9m

$2x+1$

$$x(2x+1) = 36$$
$$2x^2 + x - 36 = 0$$

Use Quadratic Formula to Solve.

$a = 2 \quad b = 1 \quad c = -36$

$$b^2 - 4ac = 1^2 - 4(2)(-36) = 289$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{289}}{2(2)} = \frac{-1 \pm 17}{4}$$
$$x = \frac{-1+17}{4} = \frac{16}{4} = 4 \checkmark \qquad x = \frac{-1-17}{4} = \frac{-18}{4} = -\frac{9}{2} \times$$

Class QZ 9

Solve $2x^2 + 5 = -2x$ by using the quadratic formula.

$2x^2 + 5 = -2x \Rightarrow 2x^2 + 5 + 2x = 0 \Rightarrow 2x^2 + 2x + 5 = 0$

$a = 2, b = 2, c = 5 \qquad b^2 - 4ac = 2^2 - 4(2)(5) = -36$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{-36}}{2(2)} = \frac{-2 \pm 6i}{4} = \frac{-2}{4} \pm \frac{6}{4}i$$

$$x = \frac{-1}{2} \pm \frac{3}{2}i \Rightarrow \left\{ \frac{-1}{2} \pm \frac{3}{2}i \right\}$$